Physics 607 Exam 1

Please be well-organized, and show all significant steps clearly in all problems.

You are graded on your work, which must be clear and legible!

An answer, even if correct, will receive zero credit unless it is obtained via the work shown.

The variables have their usual meanings: $E =$ energy, $S =$ entropy, $V =$ volume, $N =$ number of particles, $T =$ temperature, $P =$ pressure, $\mu =$ chemical potential, $B =$ applied magnetic field, $C_V =$ heat capacity at constant volume, $C_P =$ heat capacity at constant pressure, $F =$ Helmholtz free energy, $G =$ Gibbs free energy, $k =$ Boltzmann constant, $h =$ Planck constant, $c =$ speed of light. Also, $\langle \cdots \rangle$ represents an average.

In working these problems, you may assume (when appropriate) the following:

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \qquad \int_{-\infty}^{\infty} x^2 e^{-x^2} dx = \frac{1}{2}\sqrt{\pi} \qquad \int_0^{\infty} e^{-x} x^{z-1} dx = \Gamma(z) \quad , \quad \Gamma(\nu+1) = \nu\Gamma(\nu)$$

$$\langle E \rangle = kT^2 \frac{\partial}{\partial T} \ln Z \qquad C_V = -T\left(\frac{\partial^2 F}{\partial T^2}\right)_V \qquad Z = \sum_r e^{-\beta E_r} \qquad \mathbb{Z} = \sum_{Nr} e^{-\beta E_{Nr}} e^{-\gamma N}$$

$$F = -kT \ln Z \qquad \Omega = -kT \ln \mathbb{Z} \qquad Z = \frac{1}{N!}\int \frac{dq\, dp}{h^{dN}} e^{-H(p,q)/kT} \qquad \left\langle x_i \frac{\partial H}{\partial x_j} \right\rangle = \delta_{ij}\, kT$$

$$Z = \frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N \qquad \lambda_{th} = \frac{h}{(2\pi mkT)^{1/2}} \qquad \ln N! \approx N \ln N - N \qquad C_V = k\frac{x^2 e^x}{(e^x-1)^2} \;,\; x = \frac{\hbar\omega}{kT}$$

$$\int_0^{\infty} \frac{x^n}{e^x+1} dx = \left(1 - \frac{1}{2^n}\right)\int_0^{\infty} \frac{x^n}{e^x-1} dx \qquad \int_0^{\infty} \frac{x^2}{e^x-1} dx = 2\varsigma(3) \approx 2.404 \qquad \int_0^{\infty} \frac{x^3}{e^x-1} dx = 6\varsigma(4) = \frac{\pi^4}{15}$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \quad , \quad \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - .. \quad , \quad 1 + x^2 + x^3 + ... = \frac{1}{1-x}$$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \;,\; \kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \;,\; \beta = \frac{1}{kT} \;,\; \lambda = e^{\beta\mu} \;,\; \Omega = -kT \ln \mathbb{Z} = \pm kT \sum_k \ln\left(1 \pm \lambda e^{-\beta\varepsilon_k}\right)$$

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1. (a) (15) For a general thermodynamic system, show that

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V \ .$$

Hint: Consider the Maxwell relation that results from

$$dF = -SdT - PdV + \mu dN \ .$$

(b) (5) What does this imply for the dependence of the heat capacity on the volume for a classical ideal gas? (Justify your conclusion, of course.)

(c) (5) Does the same conclusion hold for a quantum ideal gas? (Justify your conclusion, of course.)

2. (20) The sound velocity in a fluid is determined in part by the adiabatic compressibility κ_S. Starting with the definitions of κ_S, the isothermal compressibility κ_T,the heat capacity at constant volume C_V, and the heat capacity at constant pressure C_P, obtain a relation between the ratios κ_S/κ_T and C_P/C_V.

[Hint: This is related to a homework problem.]

3. A classical system in D spatial dimensions, at temperature T, consists of N distinguishable particles (labeled by ℓ) with the Hamiltonian

$$\mathcal{H} = \sum_{\ell=1}^{N} \left(a_\ell \, p_\ell^r + b_\ell \, q_\ell^s \right) \ .$$

Here p_ℓ and q_ℓ are the magnitudes of the momentum and coordinate vectors in D dimensions:

$$p_\ell = \left(p_{\ell,1}^2 + p_{\ell,2}^2 + \cdots + p_{\ell,D}^2 \right)^{1/2}$$
$$q_\ell = \left(q_{\ell,1}^2 + q_{\ell,2}^2 + \cdots + q_{\ell,D}^2 \right)^{1/2} \ .$$

Also, r, s, the a_ℓ, and the b_ℓ are constants (with r and s being positive integers). The momenta and coordinates range from $-\infty$ to $+\infty$.

Please obtain your answers below in the cleanest nicest form.

You will have to give these answers in terms of definite integrals $I(D, r)$ and $I(D, s)$ which could in principle be calculated to obtain **dimensionless numbers** – but which you do not have to calculate.

Instead, just clearly define these integrals, and express your answers in terms of them.

(a) (15) Calculate the (classical canonical) partition function Z for this system (giving your answers in terms of the **dimensionless definite integrals** $I(D, r)$ and $I(D, s)$).

(b) (5) **Using your partition function**, calculate the thermodynamic energy E and heat capacity C_V.

(c) (10) Now use the **equipartition theorem** to calculate E and C_V.

(d) (5) **Use your answer to part (b)** to obtain the heat capacity for the special case of N harmonic oscillators in 3 dimensions.

4. (20) Let us assume a situation with chemical (and thermal) equilibrium at temperature T for the reaction

$$e^+ + e^- \rightleftarrows \gamma \tag{1}$$

in which an electron and positron annihilate to create a photon (in the forward reaction).

Also, with

$$T \ll mc^2 \quad , \quad \lambda_{th}^3 \ll v \equiv \frac{V}{N} \ , \tag{2}$$

the electrons and positrons may be treated as nonrelativistic classical systems.

But the rest mass energy must be included in the chemical potential. I.e., for each particle with nonzero mass m the chemical potential is

$$\mu = \mu_{gas} + mc^2 \tag{3}$$

where μ_{gas} is the usual chemical potential for a particle of mass m at temperature T in a classical nonrelativistic ideal gas.

Since matter dominates antimatter in the universe (for a currently unknown reason), assume more electrons than positrons, with

$$n^- = n^+ + n_0 \tag{4}$$

where $n^- = N^-/V$ is the number density of electrons and $n^+ = N^+/V$ the number density of positrons.

Calculate the equilibrium concentrations of positrons and electrons

$$n^+ \text{ and } n^- \quad \text{or} \quad [e^+] \text{ and } [e^-] \tag{5}$$

in terms of n_0, m, c, h, and T.

5. (COVID extra credit, 20) For a quantum ideal gas of fermions, one obtains

$$\rho \equiv \frac{N}{V} = \frac{s}{\lambda_{th}^3} f_{3/2}(\lambda)$$
$$\frac{P}{kT} = \frac{s}{\lambda_{th}^3} f_{5/2}(\lambda)$$
$$f_n(\lambda) \equiv \sum_{l=1}^{\infty} (-1)^{l-1} \frac{\lambda^l}{l^n}$$

where $s = 2$ for spin 1/2 particles. Starting with the above equations, and assuming the Taylor series expansion

$$\lambda = a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + \dots \ ,$$

obtain the quantum virial expansion up to **third** order. I.e., calculate the coefficients c_1, c_2, and c_3 in

$$\frac{P}{kT} = c_1\rho + c_2\rho^2 + c_3\rho^3 + \dots \ .$$

Stay safe!

Physics 607 Exam 2

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The variables have their usual meanings: $E=$ energy, $S=$ entropy, $V=$ volume, $N=$ number of particles, $T=$ temperature, $P=$ pressure, $\mu=$ chemical potential, $B=$ applied magnetic field, $C_V=$ heat capacity at constant volume, $C_P=$ heat capacity at constant pressure, $F=$ Helmholtz free energy, $G=$ Gibbs free energy, $k=$ Boltzmann constant, $h=$ Planck constant, $c=$ speed of light. Also, $\langle\cdots\rangle$ represents an average.

In working these problems, you may assume (when appropriate) the following:

$$\int_{-\infty}^{\infty} e^{-x^2}dx=\sqrt{\pi} \qquad \int_{-\infty}^{\infty} x^2 e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}$$

$$\int_0^{\infty} e^{-x}x^{z-1}dx=\Gamma(z) \quad , \quad \Gamma(v+1)=v\Gamma(v) \quad , \quad \Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

$$\langle E\rangle=kT^2\frac{\partial}{\partial T}\ln Z \qquad C_V=-T\left(\frac{\partial^2 F}{\partial T^2}\right)_V \qquad Z=\sum_r e^{-\beta E_r} \qquad \mathbb{Z}=\sum_{Nr} e^{-\beta E_{Nr}}e^{-\gamma N}$$

$$F=-kT\ln Z \qquad \Omega=-kT\ln\mathbb{Z} \qquad Z=\frac{1}{N!}\int\frac{dq\,dp}{h^{dN}}e^{-H(p,q)/kT} \qquad \left\langle x_i\frac{\partial H}{\partial x_j}\right\rangle=\delta_{ij}kT$$

$$Z=\frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N \qquad \lambda_{th}=\frac{h}{(2\pi mkT)^{1/2}} \qquad \ln N!\approx N\ln N-N \qquad C_V=k\frac{x^2e^x}{(e^x-1)^2}\ ,\ x=\frac{\hbar\omega}{kT}$$

$$\int_0^{\infty}\frac{x^n}{e^x+1}dx=\left(1-\frac{1}{2^n}\right)\int_0^{\infty}\frac{x^n}{e^x-1}dx \qquad \int_0^{\infty}\frac{x^2}{e^x-1}dx=2\varsigma(3)\approx 2.404 \qquad \int_0^{\infty}\frac{x^3}{e^x-1}dx=6\varsigma(4)=\frac{\pi^4}{15}$$

$$\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x\left(\frac{\partial z}{\partial x}\right)_y=-1 \quad , \quad \ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-.. \quad , \quad 1+x^2+x^3+...=\frac{1}{1-x}$$

$$\alpha=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \ ,\ \kappa_T=-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \ ,\ \beta=\frac{1}{kT} \ ,\ \lambda=e^{\beta\mu} \ ,\ \Omega=-kT\ln\mathbb{Z}=\pm kT\sum_k\ln\left(1\pm\lambda e^{-\beta\varepsilon_k}\right)$$

Be sure to note the statements and formulas on the Front Page for this exam.

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1. Consider a nonrelativistic (quantum) ideal gas of spinless bosons in 3 dimensions. There are N particles of mass m, in a volume V.

(a) (5) Calculate the density of states function $\rho(\varepsilon)$ in terms of m, V, and fundamental constants.

(b) (20) The energy is

$$E = \int_0^\infty d\varepsilon \, \rho(\varepsilon) \, \varepsilon \, n(\varepsilon) \tag{1}$$

where $n(\varepsilon)$ is the Bose-Einstein distribution function at temperature T. Starting with this expression, and showing each important step clearly, obtain the alternative form

$$E = f(T, V) \sum_{\ell=1}^{\infty} \frac{\lambda^\ell}{\ell^\nu} \tag{2}$$

where you will determine the function $f(T, V)$ and the constant ν.

Note: Do not use a relation between $\boldsymbol{E}$ and any other quantity for which a similar expansion may have been obtained. Instead show the detailed steps required to get from the integral of Eq. (1) to the sum of Eq. (2).

2. Now consider a nonrelativistic (quantum) ideal gas of spin 1/2 fermions in 3 dimensions, such as electrons in a metal. There are N particles of mass m, in a volume V. You may assume that the density of states is

$$\rho(\varepsilon) = AV\varepsilon^{1/2} \quad , \quad A = \text{ constant and } \; V = \text{volume} \; . \tag{3}$$

This system is in its ground state, at $T = 0$.

(a) (4) Calculate the Fermi energy ε_F.

Here and below, give each answer in terms of A, N, V, m, and constants.

(b) (4) Calculate the Fermi velocity v_F.

(c) (4) Calculate the total energy E.

(d) (4) Calculate the pressure P.

(e) (6) Calculate the adiabatic compressibility κ_S.

(f) (4) Calculate the sound velocity

$$v_s = \sqrt{\frac{1}{\kappa_S \, \rho_m}} \quad , \quad \text{where } \rho_m \text{ is the mass density} \; .$$

(g) (4) Show that $v_s = \text{constant} \times v_F$, where you will determine the constant.

Note: Do not use related results obtained for fermions at nonzero (or zero) temperature. Instead give the detailed steps above specifically for fermions at zero temperature.

3. (25) Let us return to the same kind of system as in problem 2 – a nonrelativistic (quantum) ideal gas of spin 1/2 fermions in 3 dimensions. There are again N particles of mass m, in a volume V, and the system is in its ground state, at $T = 0$.

Now, in addition, each particle has a magnetic dipole moment m_0. (The particles can be electrons in a metal, neutrons in a neutron star, etc. – since the neutron has a magnetic dipole moment.) We wish to obtain the magnetic susceptibility – or equivalently, the magnetic dipole moment of the system when placed in a uniform magnetic field $\boldsymbol{B}$.

Let $N_\uparrow$ and $N_\downarrow$ respectively be the number of particles with magnetic dipole moments aligned with and against the magnetic field. The total energy of an $\uparrow$ particle is changed from ε (with no field) to $\varepsilon - m_0 B$ (with the field). At $T = 0$, for fermions, the maximum total energy for these particles is just the chemical potential $\mu = \varepsilon_{max} - m_0 B$, so the maximum value of the kinetic energy ε is $\varepsilon_{max} = \mu + m_0 B$:

$$N_\uparrow = \int_0^{\mu + m_0 B} d\varepsilon \, \frac{1}{2} \rho(\varepsilon) \quad , \quad \frac{1}{2} \rho(\varepsilon) = \text{ density of states for one spin.}$$

Write the corresponding expression for $N_\downarrow$.

You may assume below that $m_0 B \ll \mu \approx \varepsilon_F$, as is true in most physical contexts, with $\rho(\varepsilon)$ smoothly varying (making $\rho(\varepsilon_F)$ most relevant).

Obtain the magnetic dipole moment M of the system, its magnetization $\mathcal{M} = M/V$, and the susceptibility

$$\chi = \frac{\partial \mathcal{M}}{\partial B}$$

to lowest order, in terms of the density of states at the Fermi surface $\rho(\varepsilon_F)$.

Note: Use the approach above, for a general system with a general density of states $\rho(\varepsilon)$, rather than using an argument or results from class notes, the textbook, etc.

4 (a) (16) Go back to the BCS theory and retrace its steps for a **2-dimensional** rather than 3-dimensional metal.

Which steps are valid (after the number of coordinates and momentum components is just changed from 3 to 2, with a 2-dimensional density of states, etc.), and which steps are invalid (after this straightforward modification)?

With the same assumptions and approximations, does the BCS theory predict a phase transition in 2 dimensions?

(b) (4) The Mermin-Wagner theorem demonstrates that conventional long-range order (such as that described by the order parameter Δ of superconductivity) cannot exist above exactly $T = 0$ in 2 dimensions.

Is your answer to part (a) consistent with this theorem? Explain.

5. (5 extra credit) Describe 3 incorrect statements about white dwarfs in the textbook.

Stay safe and sane!

(According to today's New York Times, there are now more than 2000 Covid cases at Texas A&M.)

Physics 607 Final Exam

Please be well-organized, and show all significant steps clearly in all problems.

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The variables have their usual meanings: E = energy, S = entropy, V = volume, N = number of particles, T = temperature, P = pressure, μ = chemical potential, B = applied magnetic field, C_V = heat capacity at constant volume, C_P = heat capacity at constant pressure, F = Helmholtz free energy, G = Gibbs free energy, k = Boltzmann constant, h = Planck constant, c = speed of light. Also, $\langle \cdots \rangle$ represents an average.

In working these problems, you may assume (when appropriate) the following:

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$$\int_0^{\infty} e^{-x} x^{z-1} dx = \Gamma(z) \quad , \quad \Gamma(v+1) = v\Gamma(v) \quad , \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\langle E \rangle = kT^2 \frac{\partial}{\partial T} \ln Z \qquad C_V = -T\left(\frac{\partial^2 F}{\partial T^2}\right)_V \qquad Z = \sum_r e^{-\beta E_r} \qquad \mathbb{Z} = \sum_{Nr} e^{-\beta E_{Nr}} e^{-\gamma N}$$

$$F = -kT \ln Z \qquad \Omega = -kT \ln \mathbb{Z} \qquad Z = \frac{1}{N!}\int \frac{dq\, dp}{h^{dN}} e^{-H(p,q)/kT} \qquad \left\langle x_i \frac{\partial H}{\partial x_j} \right\rangle = \delta_{ij} kT$$

$$Z = \frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N \qquad \lambda_{th} = \frac{h}{(2\pi mkT)^{1/2}} \qquad \ln N! \approx N \ln N - N \qquad C_V = k\frac{x^2 e^x}{(e^x - 1)^2} \ , \ x = \frac{\hbar\omega}{kT}$$

$$\int_0^{\infty} \frac{x^n}{e^x + 1} dx = \left(1 - \frac{1}{2^n}\right)\int_0^{\infty} \frac{x^n}{e^x - 1} dx \qquad \int_0^{\infty} \frac{x^2}{e^x - 1} dx = 2\varsigma(3) \approx 2.404 \qquad \int_0^{\infty} \frac{x^3}{e^x - 1} dx = 6\varsigma(4) = \frac{\pi^4}{15}$$

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$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \ , \ \kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \ , \ \beta = \frac{1}{kT} \ , \ \lambda = e^{\beta\mu} \ , \ \Omega = -kT \ln \mathbb{Z} = \pm kT \sum_k \ln\left(1 \pm \lambda e^{-\beta\varepsilon_k}\right)$$

Note the statements and formulas on the Front Page for this exam.

If a result has already been obtained in class - i.e. in the notes labeled "Statistical Mechanics Notes" – you can simply quote it.

Otherwise results need to be derived in your solution.

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1. (20) At one point in the early universe, when the temperature was well above 10^{15} K, all the particles of the Standard Model of particle physics can be treated as if they have zero mass. (We will leave out other hypothetical particles here.)

There are 6 quarks and 6 antiquarks, each with 2 spin states and 3 color states; 3 electron-like leptons and their 3 antiparticles, each with 2 spin states; 3 neutrinos, each with two spin states (and each its own antiparticle); 12 gauge bosons, each with two spin states (and with antiparticles included); and the Higgs boson plus its antiparticle, each with only one spin state.

Quarks and leptons (including neutrinos) are fermions, and only the quarks have distinct color states. (In taking neutrinos to be their own antiparticles, we are anticipating a successful neutrinoless double beta decay experiment,
https://en.wikipedia.org/wiki/Neutrinoless_double_beta_decay .)

Show that the total energy per unit volume, E/V, is related to the the temperature T by

$$\frac{E}{V} = \text{constant} \times T^n$$

where you will obtain the constant and n.

2. **You may quote any results from the class notes in doing this problem – but please be clear and precise in doing so.**

For a 3-dimensional system at low temperature, we obtained the Debye result $C_V^{vib} \propto T^3$ for the vibrational heat capacity and the Pauli result $C_V^{elec} \propto T$ for the electronic heat capacity.

However, there are real physical systems which behave as if they were 2- or 1-dimensional, and models relevant to real physics in higher dimensions.

(a) (10) Outlining the detailed calculation, and clearly indicating all the important steps, obtain the corresponding result in D dimensions for the vibrational heat capacity.

I.e., show through a complete argument that

$$C_V^{vib} = \text{constant} \times T^{n_1} \qquad \text{in } D \text{ dimensions}$$

where you will obtain n_1 (in terms of D).

(b) (10) Outlining the detailed calculation, and clearly indicating all the important steps, obtain the corresponding result in D dimensions for the electronic heat capacity.

I.e., show through a complete argument that

$$C_V^{elec} = \text{constant} \times T^{n_2} \qquad \text{in } D \text{ dimensions}$$

where you will obtain n_2.

3. N weakly-coupled distinguishable particles may each exist in any of 3 states with energies $-\varepsilon$, 0, and $+\varepsilon$. This system may be treated classically (i.e. with Maxwell-Boltzmann statistics), and it is in contact with a heat reservoir at temperature T.

(a) (1) What is the entropy of the system at $T = 0$?

(b) (2) What is the minimum possible energy of the system?

(c) (3) What is the maximum possible entropy of the system?

(d) (3) What is the partition function of the system at temperature T?

(e) (3) What is the most probable energy of the system at temperature T?

(f) (3) If $C(T)$ is the heat capacity of the system, what is the value of

$$\int_0^\infty \frac{C(T)}{T} dT \quad ?$$

4. Consider a 3-dimensional classical ideal gas of N_{gas} atoms, each with mass m, in a volume V at temperature T.

It is in thermal equilibrium with a 2-dimensional classical ideal gas of N_{ads} atoms, also with mass m, adsorbed on a surface of area A. The adsorbed atoms are bound to the surface with an energy ε_0.

The entire system is nonrelativistic.

(a) (5) Obtain the chemical potential μ_{gas} of the atoms in the 3-dimensional ideal gas, in terms of T, V, N_{gas}, k, m, and Planck's constant h.

(b) (10) Obtain the chemical potential μ_{ads} of the atoms in the 2-dimensional ideal gas, in terms of T, A, N_{ads}, k, m, h, and ε_0.

(c) (10) Calculate the surface coverage

$$\frac{N_{ads}}{A}$$

at a given pressure P for the gas, in terms of P, T, ε_0, and the other constants.

5. (10) **You may quote any results from the class notes in doing this problem – but please be clear and precise in doing so.**

We treated white dwarfs and neutron stars. Now imagine a hypothetical scenario in which there are stable fermions which are not their own antiparticles and which have a mass $m_d = 1000\, m_n$, where m_n is the mass of a neutron.

Using the same approach that we used earlier (with the same approximations), estimate the limiting mass M_0 and typical radius R for a "star" composed of these particles with mass m_d. Express the limiting mass as a multiple of the solar mass $M_\odot$ and the typical radius in km.

6. (10) **You may quote any results from the class notes in doing this problem – but please be clear and precise in doing so.**

For a given fluid and well-controlled experimental setup on Earth (including a fixed temperature difference between upper and lower plates), the Rayleigh-Bénard instability sets in (with smooth boundaries and cylindrical rolling) when the separation between the plates is 5.00 cm.

What is the corresponding critical value of this separation in an identical experiment on the Moon, where the acceleration of gravity is 0.166 times that at the surface of the Earth?

(You are allowed a calculator.)

The following problems are based on student talks.

7. (5 extra credit – precision and clarity required for credit) What does the Mermin-Wagner theorem tell us about ordered phases? Is it consistent with predictions of the Weiss model of ferromagnetism? Is it consistent with the Kosterlitz-Thouless transition,
https://www.nobelprize.org/uploads/2018/06/popular-physicsprize2016.pdf
https://en.wikipedia.org/wiki/Kosterlitz%E2%80%93Thouless_transition ?
Explain clearly.

8. (5 extra credit – precision and clarity required for credit) In Raman scattering, the anti-Stokes light emitted by the material or molecule is at a higher frequency ω' than the frequency ω of the incident light, because a phonon has been absorbed: $\hbar\omega' = \hbar\omega + \hbar\omega_{phonon}$. What are **two** ways in which Raman scattering can measure temperature? Also, why would one want to use this technique? Explain clearly.

Please stay safe and sane, and enjoy the holidays!