PHYS 206

Brush up your maths!

"The only way to learn *mathematics* is to do *mathematics*"- Paul R. Halmos

Q1. Solve for x: x/a + x = 8b

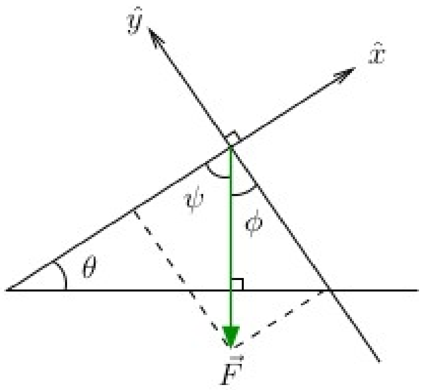
(A) 4ab
(B) 8ab
(C) 8b/ (a-1)
(D) 8b/(a+1)
(E) 8ab/(1+a)

Q2. If 2x + 2y = 0 and 2x − 2y = −4, then

(A) x = 0, y = −2
(B) x = −2, y = 2
(C) x = 0, y = 2
(D) x = −1, y = 1
(E) x = 2, y = −2

Q3. In the right-hand diagram, $\theta = 30.0^\circ$, what is the value of ϕ ?

(A) $\Phi = 45^\circ$
(B) $\Phi = 60^\circ$
(C) $\Phi = 30^\circ$
(D) $\Phi = 90^\circ$
(E) $\Phi = 120^\circ$



Q4. In the diagram above, $\theta = 30.0^\circ$, and the vector **F** is 20.0 units long. Given that $\cos 30^\circ = \sqrt{3}/2 \approx 0.866$ and $\sin 30^\circ = 1/2$, this means that (there may be more than one correct answer, but choose only one):

(A) $\mathbf{F}_x$ is 10.0 units long
(B) $\mathbf{F}_y$ is 34.6 units long
(C) $\mathbf{F}_x$ is 8.7 units long
(D) $\mathbf{F}_y$ is 17.3 units long
(E) $\mathbf{F}_y$ is 11.5 units long

Q5. If a sphere has a radius of 0.50 m, then the surface area of the sphere is

(A) 6.28 m^2
(B) 0.32 m^2
(C) 12.6 m^2
(D) 3.14 m^2
(E) 0.16 m^2

PHYS 206 Don't Panic: Motion in 1D, Vecors!

"Vector! That's me. Because I'm commiting crimes with both direction and magnitude! Oh yeah!" (Despicable Me)

[1] An object with mass m moves along the x-axis. It is observed to have a velocity in the plus x direction with magnitude $c_1 t^2$ where c_1 is a positive, known constant and t is the time in seconds. At t=1sec, it is observed to be at the point x=D. When does the object reach the point x=2D?

[2] Two vectors, $\vec{A}$ and $\vec{B}$, have magnitudes A = 1 and B = 5. Their vector product is $3\hat{\imath} - 4\hat{\jmath}$.

1. Determine the angle between vectors $\vec{A}$ and $\vec{B}$?
2. Find the scalar product of these two vectors?

[3] You're designing a machine that needs to stop an object, falling from rest, just before it hits a conveyor belt a height H below. The machine will slow the object down so that its new acceleration is, $|\vec{a}| = g - ct$, where c is an unknown constant. Find what c must be for the object to stop before hitting the conveyor belt.

[4] **Waking the Balrog:** In The Fellowship of the Ring, the hobbit Peregrine Took (Pippin for short) drops a rock into a well while the travelers are in the caves of Moria. This wakes a balrog (a bad thing) and causes all kinds of trouble. Pippin heard the rock hit the water 7.5 s after he dropped it.

(a) Ignoring the time it took the sound to get back up, how deep is the well?
(b) If the speed of sound is 340 m/s (it was pretty cool in that part of Moria), was it OK to ignore the time it takes sound to get back up? Discuss and support your answer with a calculation.

Extra Problems ::

1. **Race:** Blythe and Geoff compete in a 1-km race. Blythe's strategy is to run the first 600 m of the race at a constant speed of 4 m/s, and then accelerate with constant acceleration to her maximum speed of 7.5 m/s, which takes her 1 min, and finish the race at that speed. Geoff decides to accelerate with constant acceleration to his maximum speed of 8 m/s at the start of the race and to maintain that speed throughout the rest of the race. It takes Geoff 3 min to reach his maximum speed. Who wins the race?

2. **Balls Off the Roof:** A ball is thrown straight up from the edge of the roof of a building. A second ball is dropped from the roof a time t_1 later. Ignore air resistance.

(a) If the height of the building is H, what must the initial speed of the first ball be if both are to hit the ground at the same time?
(b) On a graph, sketch the positions of both balls as a function of time, measured from when the first ball is thrown.
(c) Consider the same situation, but now let the initial speed $v_0 = v_L$ of the first ball be given and treat the height h of the building as an unknown.

3. **Waking the Balrog:** In The Fellowship of the Ring, the hobbit Peregrine Took (Pippin for short) drops a rock into a well while the travelers are in the caves of Moria. This wakes a balrog (a bad thing) and causes all kinds of trouble. Pippin heard the rock hit the water 7.5 s after he dropped it.

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PHYS 206: Motion in 2D!

"If it doesn't **challange** *you, it doesn't* **change** *you!" - Fred Devito*

[1] **DP-Ex3:** An object's acceleration is given by

$$\vec{a} = \alpha t\hat{i} + \beta t^2\hat{j}$$

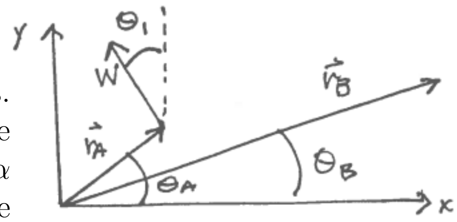
If the object starts at t=0 at the origin and initial velocity v_0 making an angle θ with x-axis, what are its velocity and position at any time?

[2] **Summer,'18:** A block of mass m is moving in the x,y plane. At t=0, it is placed at the point $\vec{r}_A$, and given the initial velocity of magnitude W with an angle θ_1 (see figure).

The block's acceleration is given by

$$\vec{a} = \alpha\hat{i} + \beta t\hat{j}$$

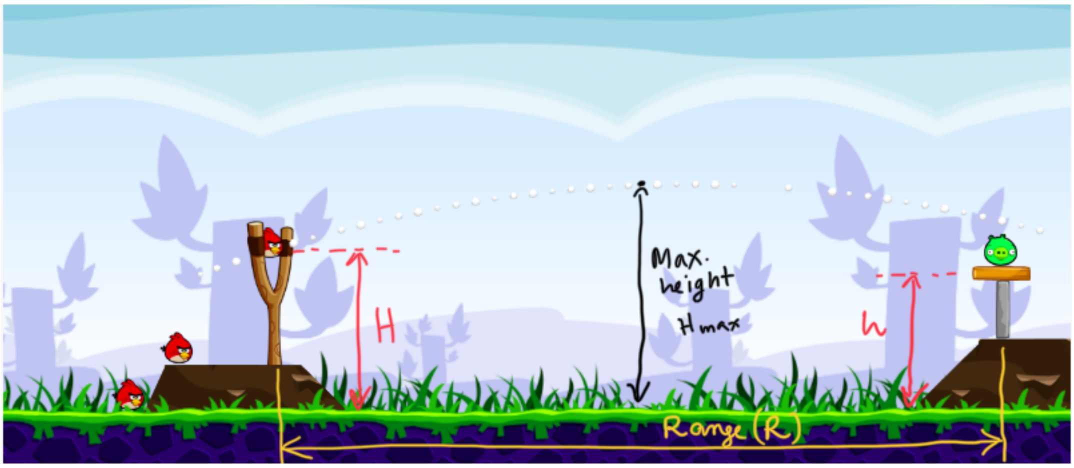
where α, β are unknown constants. The time required to reach the point $\vec{r}_B$ is given to be T. Find α and β. (The length of $\vec{r}_A$ is r_A, the length of $\vec{r}_B$ is r_B and the angles θ_A, θ_B are all known.)



[3] **DP-Ex6:** A rocket is fired from a tower of height H, at t=0, with an initial velocity of magnitude W at an angle θ above the horizontal. The rocket's engine supplies an acceleration that increases with time according to $a(t) = \alpha t^2$. The acceleration is always directed at an angle β with the horizontal. The rocket's fuel runs out after 2 seconds. What will its velocity and position be when the fuel runs out? Neglect gravity.

[4] **Maximizing range:** Show that the range of a projectile maximizes if you throw it at angle 45^o.

[5] **Angry Birds:** You are playing *Angry Birds Dream Blast*, and your bird is initially at H above the ground. To break the green pot kept at h above the ground on the other side, you release your bird from the catapult at velocity v_0, and an angle θ with respect to ground. Obtain the following expressions
(a) Maximum height (H_{max});
(b) The total time of flight (T);
(c) The distance between the catapult and the target (or range R);
(d) Consider, the two heights are same i.e. H = h, and check your answers with the results of a standard projectile motion.



[6] **Airplane:** An airplane flying upwards at v_0 and at angle θ relative to the horizontal releases a ball when is it is h above the ground. Neglect any effects due to air resistance and calculate

(a) The time of it takes the ball to hit the ground;
(b) The maximum height of the ball;
(c) The horizontal distance the ball travels from the release point to the ground.

[7] **Balloons:** A water balloon is thrown horizontally at a speed of v_0 from the roof of a building that is h above the ground. At the same instant the balloon is realized, a second balloon is thrown down at the same velocity from the same height.

(a) Determine which balloon hits the ground first.
(b) Determine how much sooner it hits the ground than the other balloon.
(c) Which Balloon is moving with the fastest speed at impact?

PHYS 206: Newton's Law!

"If it doesn't **challange** *you, it doesn't* **change** *you!" - Fred Devito*

[1] **Horizontal Blocks:** Two blocks are in contact on a horizontal, frictionless surface. Block 1 has a mass of m and block 2 has mass of M. If an external force F pushes on block 1 what is the magnitude of the force that acts on block 2?

[2] A particle of mass m is subject to two forces,$\vec{F_1} = c_1\hat{i}_x + c_2\hat{i}_y$ and $\vec{F_2} = c_3\hat{i}_x + c_4\hat{i}_y$. The particle is initially at rest at the point $x = -x_0, y = y_0$.

a. What are the components of the particle's velocity as a function of time?
b. What are the coordinates of the particle as a function of time?

[3] **Spatially varying force:** An object of mass m is at rest in equilibrium at the origin. At t = 0 a new force $\vec{F}(t)$ is applied that has components

$$F_x(t) = k_1 + k_2 y$$

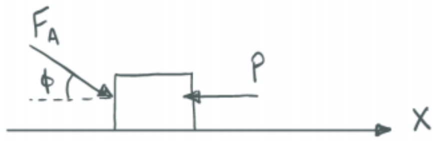
$$F_y(t) = k_3 t$$

where k_1, k_2 and k_3 are known constants. Calculate the position and velocity vectors as functions of time.

[4] The position of a block of mass m was found to be

$$x(t) = c_1 t^3 + c_2 t^2$$

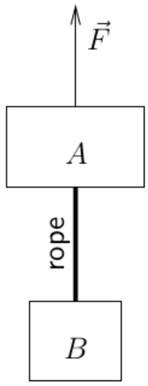
There are two forces acting on the block; P , a constant horizontal force pointing in the opposite direction of motion, and an unknown force F_A pointing at an angle ϕ as shown. Find the magnitude of the unknown force.



[5] A block of mass m is dropped, from the rest, from a height H above the ground at time t=0. In addition to gravity, a strange force $f(t) = \alpha t^2$ acts vertically upwards, where α is known constant. Also, consider a constant air resistance (force) β. Assuming that the block doesn't hit the ground, when and where will it be turns around?.

[6] A block of mass m is held stationary against a vertical frictionless wall by an applied force P . This force is applied at an angle of ϕ relative to the horizontal. Find the value of P.

[7] The two blocks depicted to the right are connected by a heavy uniform rope with a mass of m rope = 4.00 kg. The mass of block A is m A = 6.00 kg and that of block B is m B = 5.00 kg. An upward force of 180 N is applied as shown. Take g = 10 m/s^2 for this problem.



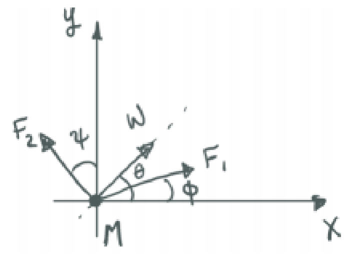
A) What is the acceleration of the system?
   (a) 8 m/s^2
   (b) 2 m/s^2
   (c) 10 m/s^2
   (d) 4 m/s^2

B) What is the tension at the top of the heavy rope? Is it the same at midpoint of the rope? If no, then what is the tension at the midpoint?
   Calculate T_{top}, T_{mid} and T_{bottom}. Check, $T_{mid} = \frac{1}{2}(T_{top} + T_{bottom})$

PHYS 206: Newton's Law (2^{nd} worksheet)!

"The law of *conservation of energy* tells us we can't get something for nothing, but we refuse to believe it." - Isaac Asimov

[1] A particle of mass M is moving in the xy-plane as shown. At t=0 the block is given an initial velocity W pointing at an angle θ with the horizontal. At that same time two forces are applied, one with magnitude F_1 pointing at a constant angle ϕ with the horizontal and the other, $F_2 = ct$ pointing at a constant angle ψ with respect to the vertical. (Neglect gravity) Find the blocks position as a function of time.



[2] **Three blocks on a table:** Three blocks of masses 1kg, 2kg and 3kg are kept together side-by-side on a frictionless table, and a 18N force is applied to the leftmost block of 1kg. What is the force acting on the 2kg block? (**For more insights, see the notes!**)

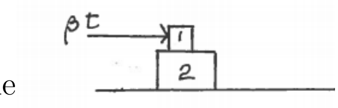
[3] **A Block on top of another:** Two blocks, of masses m_1 and m_2 , are at rest on a horizontal, frictionless surface. The smaller block, m_1 , rests on top of the larger block m_2 . A constant, horizontal force is applied to the top block such that the two blocks move together along the surface. What is the maximum horizontal force which may be applied to the top block so that the blocks move together? (**For more insights, see the notes!**)

[4] **DP-Ex9:** A small block, mass 2kg, rests on top of a larger block, mass 20kg. The coefficient of friction between the block is 0.25. If the larger block is on a frictionless table, what is the largest horizontal force that can be applied to it without the small block slipping?

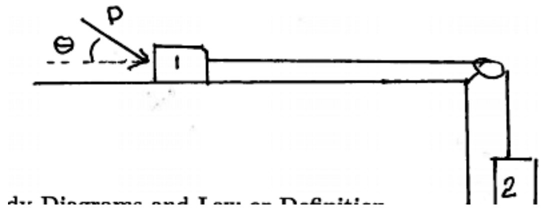
[5] **Summer,'19:** A block of mass m_1 is on top of a block of mass m_2. The coefficient of friction between the blocks is μ and the surface they move on is frictionless. At t=0, a horizontal force of magnitude βt applied to the upper block, where β is known, positive constant.

- A) Assuming the two blocks move together, find every force acting on each block in terms of the given quantities.

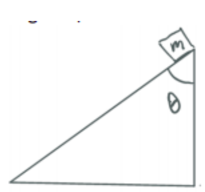
- B) At what time will the upper block begin to slip so that the two blocks will no longer move together?

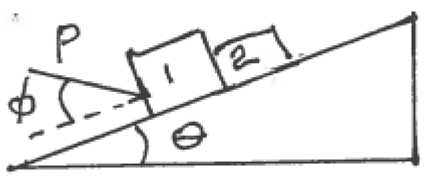


[6] **Summer,'19:** A block, mass m_1 moves to the right on a horizontal surface. It is being pulled by a massless, unstretchable rope that goes over a frictionless pulley connected to a block of mass m_2. The coefficient of friction between block 1 and the surface is μ. Another known force P acts at the known angle θ. Find the accelerations of the blocks.

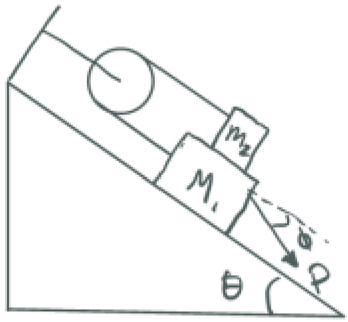


[7] **Block slipping in an incline:** A block mass m starts at rest at the top of an inclined plane of height H and angle θ as shown. There is friction between the block and plane with coefficient of friction μ . In addition to the usual forces there is also a force with magnitude $F = \beta t$ pointing down along the inclined plane.
A) When does the block first begin to slip?
B) Find the velocity of the block at the bottom of the incline plane. (No Algebra, set the equations)





[8] **Summer,'17:** Two blocks of masses m_1 and m_2, are being pushed up an inclined plane by a force of magnitude P directed at an angle ϕ as shown. The plane is inclined at the angle θ. Find the force that block 1 exerts on block 2 assuming that the only friction is between block 1 and the plane with coefficient of friction μ.

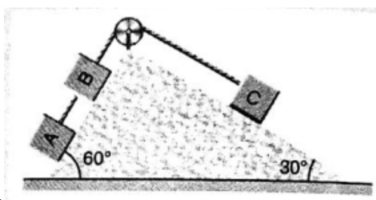


[9] Two blocks M_1 and m_2 are connected to each other by a massless rope that is then connected to a pulley. The inclined plane is frictionless and the only friction is between the two blocks with coefficient μ. An additional force with constant magnitude P is applied to the bottom block and points at an angle ϕ as shown. Find the magnitude of the tension in the rope connecting the two masses assuming the masses are not moving.

[10] **DP-Ex10 (similar):** Two blocks of mass m and M are connected by a massless, unstretchable rope that passes over a massless, frictionless peg, slide on an incline plane. Inclination angles are α and β for m and M respectively.

- A) Find the acceleration of the blocks and the tension in the rope.
- B) If there is no acceleration, find the ratio of the masses of these two blocks.

[11] **Three blocks on an incline:** In the figure, mass of blocks A, B and C are 1 kg, 3kg and 2kg respectively.

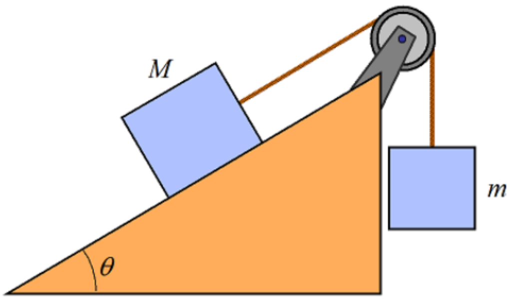


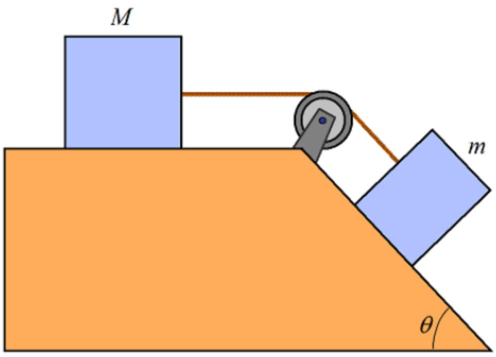
- A) The acceleration of the system.
- B) Tension in the string.

Neglect friction. Take g = 10 m/s^2, sin 30^o = cos 60^o = 0.5 and sin 60^o = cos 30^o = $\sqrt{3}/2$

Please turn over

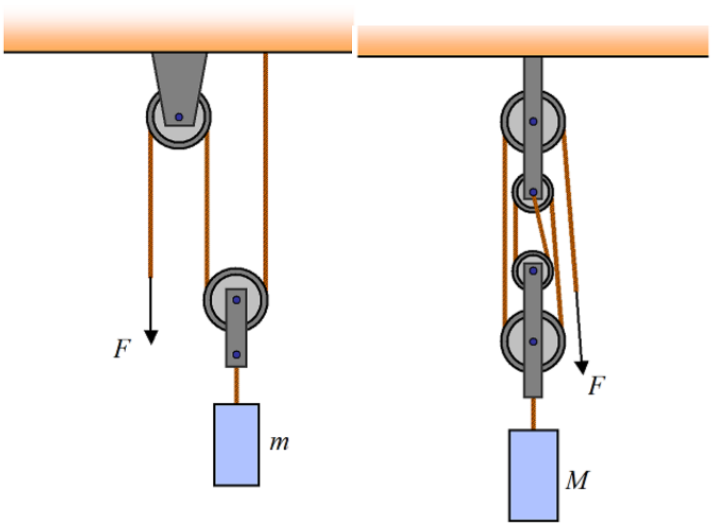
[12] **Pulley Prob1:** Two blocks of mass m and M are connected via pulley with a configuration as shown. The coefficient of static friction is μ_s, between block and surface. What is the minimum and maximum mass M so that no sliding occurs?





[13] **Pulley Prob2:** Two blocks of mass m and M are connected via pulley with a configuration as shown. The coefficient of static friction is μ_s, between blocks and surface. What is the maximum mass m so that no sliding occurs?

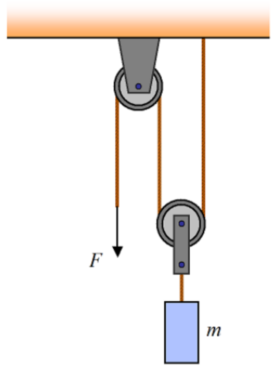
[14] **Pulley Prob3:** A block of mass m is lifted at constant velocity, via an arrangement of pulleys as shown. Determine the pulling force F for both cases. Ignore the mass of the pulleys.



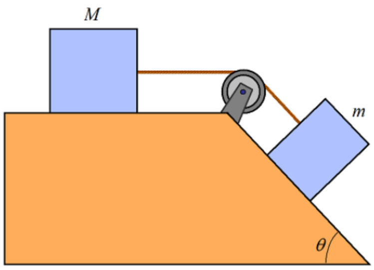
PHYS 206: Newton's law, Work-Energy!

"The law of *conservation of energy* tells us we can't get something for nothing, but we refuse to believe it." - Isaac Asimov

Newton's law ::



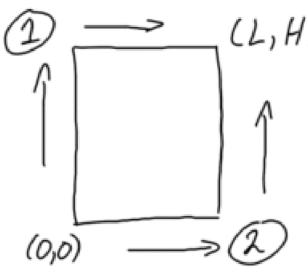
[1] **Pulley Prob1:** A block of mass m is lifted at constant velocity, via an arrangement of pulleys as shown. Determine the pulling force F. Pulleys are massless, frictionless.



[2] **Pulley Prob2:** Two blocks of mass m and M are connected via pulley with a configuration as shown. The coefficient of static friction is μ_s, between blocks and surface. What is the maximum mass m so that no sliding occurs?

Work-Energy ::

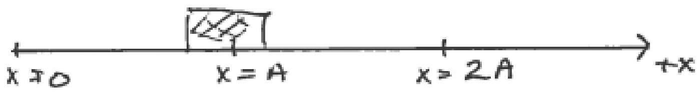
[3] **Varying force:** The force F acting on the box is given as $\vec{F} = (\alpha x - \beta x^2)\hat{i}_x - \gamma y^3 \hat{i}_y$ where α, β, γ are known constants and $\vec{r}$ is displacement in the direction of motion. Assuming that F is the only force acting on the box, find work done on the box when the box is displaced by the force from position (0,0) to position (x_0, y_0).



[4] **Path function or state function? :** An object travels from the origin to the point (L,H) via two different paths as shown. Find the total work done by each of the forces listed below. ($\alpha, \beta, \gamma, \delta$ are constants.)

- (a) $\vec{F_1} = \alpha y \hat{i} + \beta \hat{j}$
- (b) $\vec{F_2} = \gamma x \hat{i} + \delta y \hat{j}$

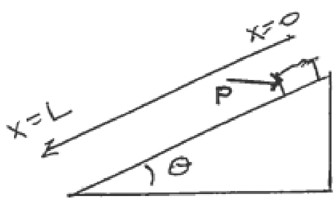
[5] **Summer,'18:** A small block of mass m on a horizontal surface starts at the point x=A with velocity v to the right. The coefficient of friction between the block and the surface is μ. A force is exerted on the block which points to the right and has a magnitude kx^2, where k is an unknown constant. What must k be if the block is to have velocity $2v$ to the right at the point x=2A?

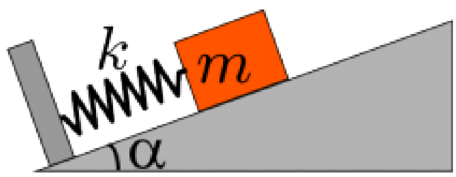


[6] An object of mass m is placed at the point x=A on a horizontal table with an initial speed of magnitude v_0 directed to the right. A horizontal force is pushing to the right with a magnitude of $F = \alpha x$. There is friction between the block and the table which has a coefficient of friction $\mu(x) = \mu_0\Big(1 + \frac{x}{x_0}\Big)$.

- (a) Where will the object stop?
- (b) Where will the object stop if instead, $F = F_0$, and $\mu(t) = \mu_0\Big(1 + \frac{t}{T}\Big)$

[7] **Summer,'19:** A box of mass m is placed at rest on an inclined plane, known angle θ. A known horizontal force, magnitude P is applied to the block. The coefficient of friction between the block and the plane is a function of the position on the plane given by $\mu = \mu_0(1 + \frac{x}{L})$. Assuming the block slides down the plane, find it's velocity at the point x=L/2.



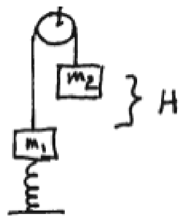


[8] **Spring:** Box of mass m is pressed against a spring on a frictionless incline. The plane is making an angle of α with respect to the horizontal surface. The spring constant is k. Spring is initially compressed on Δx. The box was released at t = 0. Find speed of the box at the moment when the spring is completely decompressed.

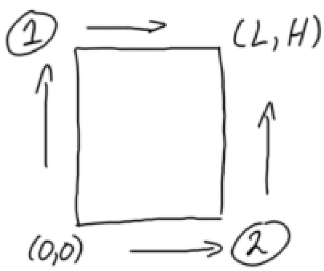
PHYS 206: Work-Energy (2^{nd} worksheet)!

"The law of *conservation of energy* tells us we can't get something for nothing, but we refuse to believe it." - Isaac Asimov

[1] Two masses are connected by a massless string, passing over a massless, frictionless pulley. Mass m_1 is connected to an ideal spring, with spring constant k, as shown. Find the speed of the masses after m_2 moves a distance of H down.



[2] **Kinetic Sculpture:** Part of artist's sculpture consists of a object with mass M (you can't tell what it is supposed to be, but it's art) and another object with mass m (m < M) which hang straight down from opposite ends of a very thin, flexible wire. This wire passes over a smooth, cylindrical, horizontal, stainless steel pipe H above the floor. The frictional force between the rod and the wire is negligible. The M object is held h_1 above the floor and the other object hangs h_2 above the floor ($h_2 < h_1$). When the mechanism releases the M object, both objects accelerate and one will eventually hit the floor, but they don't hit each other. Calculate the speed of the object which hits the floor.



[3] **Path function or state function? :** An object travels from the origin to the point (L,H) via two different paths as shown. Find the total work done by each of the forces listed below. ($\alpha, \beta, \gamma, \delta$ are constants.)

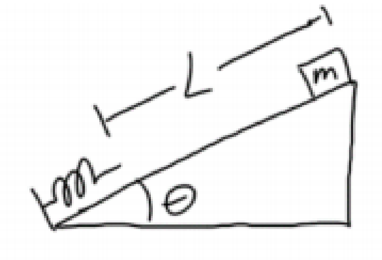
- (a) $\vec{F_1} = \alpha y\hat{i} + \beta\hat{j}$
- (b) $\vec{F_2} = \gamma x\hat{i} + \delta y\hat{j}$

[4] **DP-Ex7:** An object of mass m moves so that its position is given by

$$x(t) = c_1 t^2 + c_2 t$$

$$y(t) = c_3 t + c_4$$

with c_1, c_2, c_3 and c_4 constants. FInd the potential energy function for the force acting on it.

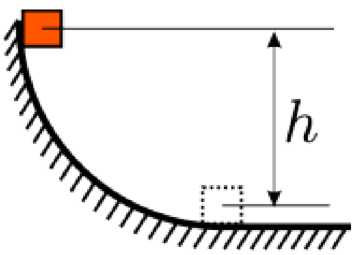


[5] A block of mass m starts at the top of an inclined plane as shown. It is gently pushed to give it an initial velocity down the incline of v_0. There is a coefficient of friction between the block and the plane of μ (assume this applies for both kinetic and static conditions). The block will move down the incline, and compress a spring, with spring constant k.

- (a) Find an equation for the maximum compression of the spring.
- (b) Find an equation which will determine whether the block will be able to rebound back up the incline.

[6] This is a one-dimensional problem. An object of mass m is acted upon by a force given by $F_x = -\beta(x - c)$, where β and c are positive known constants.

- (a) Determine whether or not this force is conservative.
- (b) If the object is placed at the point x=A and given a velocity of magnitude v_1 in the positive x-direction, what will its velocity be at the point x=A/2 if the above force is the only force acting on it?
- (c) Where will the kinetic energy have its maximum value?



[7] **Box on hemisphere:** Box with mass m is placed on rough surface. The surface has shape of 1/4 of the cylinder as it is shown on the figure. The radius of the cylinder is known and equal h. Once released, the box slides down and stops because of friction right at the bottom of the cylinder. Find work done by friction during this motion.

[8] **Friction:** Block 1 and block 2 have the same mass, m, and are released from the top of two inclined planes of the same height making 30^o and 60^o angles with the horizontal direction, respectively. If the coefficient of friction is the same in both cases, which of the blocks is going faster when it reaches the bottom of its respective incline?

PHYS 206: Work-Energy, Conservation of Energy!

"The law of *conservation of energy* tells us we can't get something for nothing, but we refuse to believe it." - Isaac Asimov

[1] **DP-Ex6:** A particle of mass m is attracted to the origin with a force of magnitude

$$|F_x| = \frac{k}{x^2}.$$

If the particle starts at x=A, moving to the right with velocity v_0, where will it turn around?

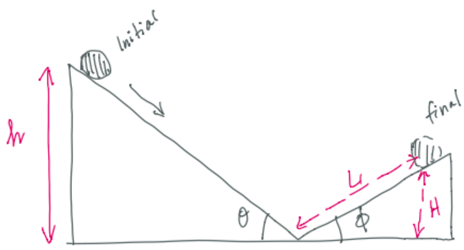
[2] The force on a particle is represented by the potential-energy

$$U(x) = -b\Big(\frac{1}{a+x} - \frac{1}{2a-x}\Big);$$

in the region -a < x < a, and both a,b ≥0.

- (a) What are SI units of a and b.
- (b) Find the force in this region.
- (c) At what value of x, the force is zero? Is this a stable or unstable point?

[3] An object is going down from the first inclined plane (with inclination angle θ) of height h (see figure), and then moving upwards in the second inclined surface (with inclination angle ϕ). Find the distance (L) and height (H) after which the object will stop [ignore friction, and energy loss in the juction of two inclined surfaces].



[4] **From Galileo to Newton:** What happens if $\phi = 0$ in the previous problem? Can you predicts *Newton's First Law* from this?

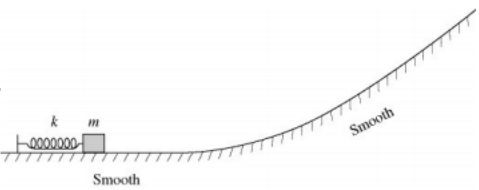
[5] **Summer,'17:** The motion of a small object of mass m is observed as it moves along the x-axis. Its kinetic energy is measured and found to vary with x according to $\frac{1}{2}mv_x^2 = c_1 + c_2x^4$ where c_1, c_2 are known positive constants. There are two forces acting on the object. One is given by $F_{1x} = c_3x^3$, where c_3 is a known constant. What is the other conservative force F_{2x}?

[6] **Summer,'17:** A small block of mass m on a horizontal surface starts at the point marked x=A with velocity v_1 to the right. The coefficient of friction between the block and the surface is a function of x given by $\mu(x) = \mu_0(1+\frac{x^2}{L^2})$, where L is a known constant. A springlike force is exerted on the block which points to the left and has magnitude kx, where k is a known constant. Where will be the block has zero velocity? (No algebra, only set up the equation)

[7] **Summer,'16:** You are going to fire a cannon with a initial velocity v_m at an angle θ with the horizontal. In addition to gravity, there is a strange height dependent force βy^2 acting in the same direction as gravity. What should be your v_m so that the cannon ball will reach a maximum height of H if the θ is known? The cannon ball has mass m.

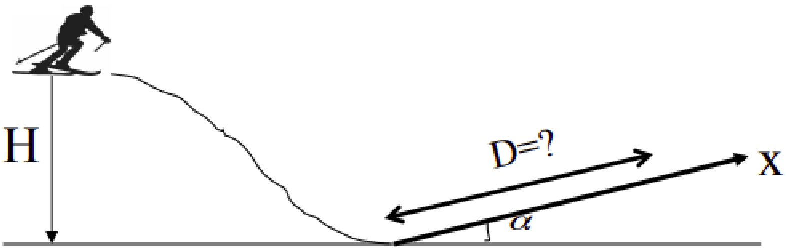
[8] **Friction:** Block 1 and block 2 have the same mass, m, and are released from the top of two inclined planes of the same height making 30^o and 60^o angles with the horizontal direction, respectively. If the coefficient of friction is the same in both cases, which of the blocks is going faster when it reaches the bottom of its respective incline?

[9] **Shooting the box:** : A box of mass m is pressed against (but is not attached to) an ideal spring of force constant k and negligible mass, compressing the spring a distance Δx. After it is released, the box slides up a frictionless incline as shown in the figure and eventually stops at height h. If we repeat this experiment but instead use a spring having force constant 2k, how high will the box go up the plane?

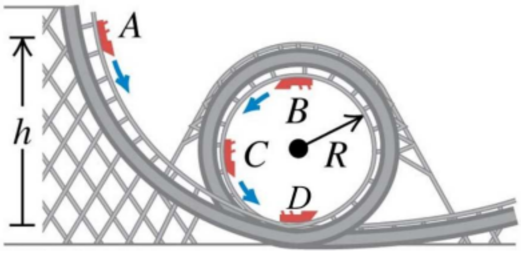


[10] **Summer,'13:** A skier with mass m starts from rest at the top of the frictionless ski slope H meters high. He immediately loses control over his skis and goes straight downhill. Fortunately, at the bottom of the slope he enters an upward ramp of constant slope angle α. The ramp has a soft snow surface with a coefficient of friction μ.

- (a) What is the distance, D, that the skier moves up the ramp before coming to a halt?
- (b) Find this distance D if the coefficient of friction is given by $\mu = \mu_0 + cx^2$ where c is a known constant and x is counted from the bottom of the ramp along its surface. Stop when you have one equation for one unknown. Don't solve it.



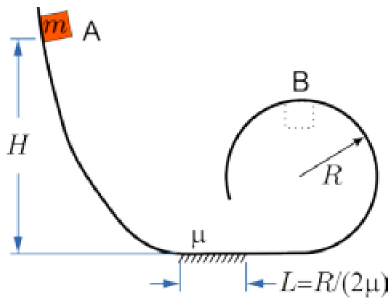
[11] At an amusement park, a car of mass m rolls without friction around a track as shown. The car starts from rest at point A, a height h = 3R above the bottom of the loop (R is the radius of the loop). Treat the car as a point-like particle.



- (a) Find the car's kinetic energy and normal force acting on the car at the **top** of the loop (point B). Normal force: upward or downward?

- (b) Find the car's kinetic energy and normal force acting on the car at the **bottom** of the loop (point D). Normal force: upward or downward?

[12] **Roller coaster:** A car of mass m is placed at the top of a roller coaster as shown below and released from rest. The whole trck is all frictionless except for a damaged part just before the circular part on the right side where the coefficient of friction is μ_k over a length $L = R/2\mu_k$, where R is the radius of the circular part. When the car reaches the top of the circular loop, it has a speed $v = \sqrt{gR}$ and does not lose contact with the track. Throughout the motion, the car slides without slipping, rolling.

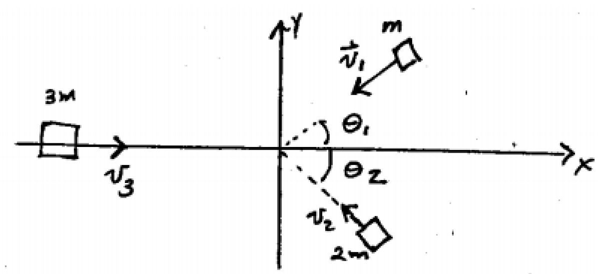


- (a) Find the work done by friction as the car crossed the rough patch before the circlular track.

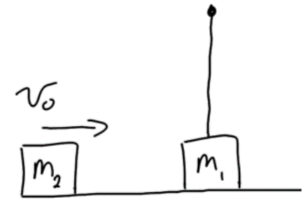
- (b) Find the height, H from which the car should be released so that it reaches the speed v at the top of the circle.

1. Two balls, masses m_1 and m_2 , are initially moving towards each other with known velocities. The velocity of the second ball is twice that of the first ball. The balls then collide, and rebound along their initial directions with the same speed as the first ball before the collision. What ratio would the masses need to have for this to be possible?

2. Three blocks are initially sliding towards each other on a frictionless table top as shown below. The first block, with mass m , has a known velocity of magnitude v_1 in the direction shown, where θ_1 is also known. The second, mass $2m$, has an unknown velocity of magnitude v_2 in the direction given by the known angle θ_2 . A third block, mass $3m$, has a velocity of known magnitude v_3 and moves along the $+x-axis$. The three blocks collide at the origin, stick together and move together along the $+x-axis$ with an unknown, post collision velocity of magnitude u .
   a. What was the initial velocity of the second block?
   b. What is the magnitude of the final velocity of the three-block system?

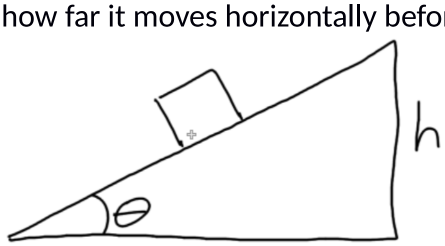


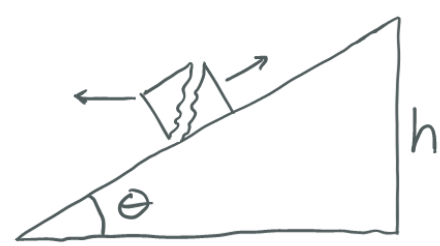
3. A block of mass m_1 is at rest on a horizontal, frictionless table. This block is connected by a light string to a fixed point directly above it, forming a pendulum, which may swing freely about this fixed point. A second block, of mass m_2 moving at a speed of v_0 impacts the first block and recoils back along its original direction at half of its original speed. Find the maximum height that m_1 rises to after the impact.



4. A block of mass M, is at rest on a horizontal table. A bullet of mass m, is shot at the block with a speed of v_B, impacting it at an angle of θ, relative to the vertical as shown. The bullet becomes embedded in the block and the two move to right afterwards towards an ideal spring of spring constant k.
   a. Assuming the table is frictionless, find the maximum compression of the spring.
   b. Find an equation for the maximum compression of the spring if there is a coefficient of friction of μ between the block and the table, and the block starts a distance of L from the spring.

5. A block of mass M is initially at rest half way up an inclined plane (inclined at an angle θ relative to the horizontal), when it suddenly explodes into two equal fragments! The first fragment moves up the incline plane (and experiences friction with coefficient of μ). The second fragment moves off horizontally as shown. If the first fragment comes to rest at the very top of the incline plane, find the speed of the second fragment. Can you determine how far it moves horizontally before it lands?





PHYS 206: Rotational Dynamics!

"The law of *conservation of energy* tells us we can't get something for nothing, but we refuse to believe it." - Isaac Asimov

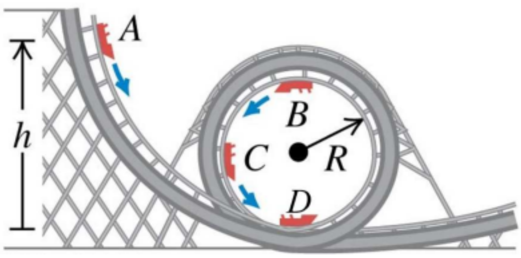
:: Circular Motion ::

[1] **Summer,'19:** In another galaxy,far, far away, a small moon, mass m, travels in a circle of radius r about the fixed Sun. The force exerted by the Sun on the moon is not the usual force of gravity. It is attractive and has magnitude

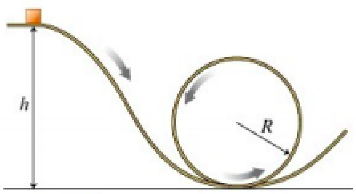
$$|\vec{F}| = c\frac{m}{r^4}$$

where c is a constant and r is the distance between the Sun and the moon. Find the total energy for the moon as a function of r.

[2] **Roller coaster 1:** At an amusement park, a car of mass m rolls without friction around a track as shown. The car starts from rest at point A, a height h = 3R above the bottom of the loop (R is the radius of the loop). Treat the car as a point-like particle.

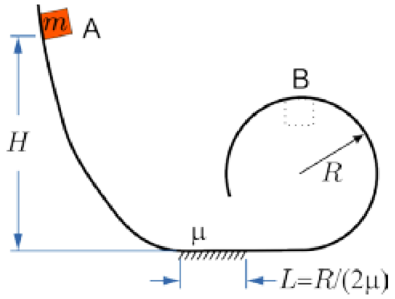


- (a) Find the car's kinetic energy and normal force acting on the car at the **top** of the loop (point B). Normal force: upward or downward?
- (b) Find the car's kinetic energy and normal force acting on the car at the **bottom** of the loop (point D). Normal force: upward or downward?



[3] **Roller coaster 2:** A frictionless track contains a circular section of radius R as shown.

- (a) If the object is released from rest, what is the minimum height at which a block must be started in order for it to go around the loop without falling off?
- (b) If instead the object launched horizontally from the ground (H=0) towards the loop, what is the minimum speed it would need to go around the loop without falling off?



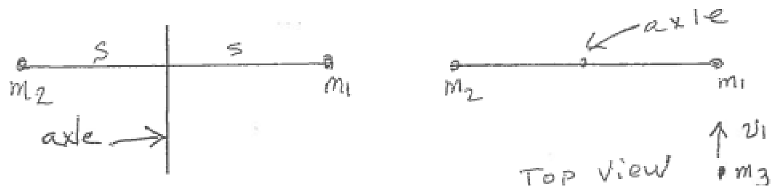
[4] **Roller coaster 3:** A car of mass m is placed at the top of a roller coaster as shown below and released from rest. The whole trck is all frictionless except for a damaged part just before the circular part on the right side where the coefficient of friction is μ_k over a length $L = R/2\mu_k$, where R is the radius of the circular part. When the car reaches the top of the circular loop, it has a speed $v = \sqrt{gR}$ and does not lose contact with the track. Throughout the motion, the car slides without slipping, rolling.

- (a) Find the work done by friction as the car crossed the rough patch before the circlular track.
- (b) Find the height, H from which the car should be released so that it reaches the speed v at the top of the circle.

:: Torque, MOI, and Conservation of Angular Momentum ::

[5] **DP-Ex2:** If $\vec{r} = x\hat{i} + y\hat{j}$, and $\vec{p} = p_x\hat{i} + p_y\hat{j}$; find the components of $\vec{L}$.

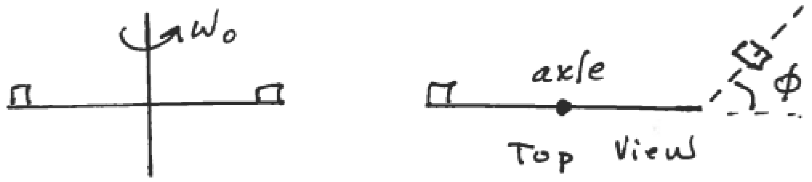
[6] **Summer,'17:** Two blocks, mass m_1, m_2 are connected by a massless, rigid rod. The rod is connected to a vertical axle which is free to rotate. Each mass is a distance S from the axle. The system is initially at rest. A small block of mass m_3 strikes one of the blocks and sticks to it. It was travelling horizontally with velocity v_1 in the direction shown, perpendicular to the rod (ignore gravity).What will be the angular velocity of the system after the collision?



Note: Do the following problems.

- **Chap 14: Problems + Ex-3,4,5,6**
- **Chap 15: Problems + Ex-2,3,4,6,7**
- **Read the Chap 16 for review of all of these things.**
- **Old exam papers.**

[7] **Summer,'16:** A rod of length S has moment of inertia I about its center. The rod is free to rotate about a vertical axle through its center. Two small blocks, each of mass m are glued to the rod at the positions shown. The entire system is set rotating about the axle with angular velocity ω_0. At some intant one of the blocks comes loose and flies off with a velocity of magnitude v_0 at the angle ϕ shown below. What will be the angular velocity of the remaining block?



[8] **Merry-go-round:** A small child of mass m , stands at the center of a massless merry-go-round. The child then begins to walk radially outward from the center of the merry-go-round so that their position is given by $r(t) = c_1 t$. At all times the system is held at a constant angular velocity of ω_0.

- (a) Find the net radial and angular forces acting on the child.
- (b) Find the torque exerted on the child.

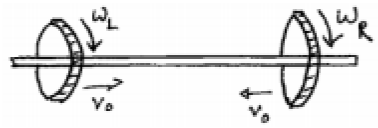
[9],[10] Solve the following problems: **DP-Ex3 and Ex-6**.

[11] **Summer,'18:**

In a famous Physics 218 experiment a student stands on a platform which is free to rotate on frictionless bearings. The moment of inertia of the platform about the vertical axle through its center is a known constant *I*. He has his arms extended with a huge mass *m* in each hand. If he is set into rotation with angular velocity ω_0 and then drops his hands to his sides, what happens to his angular velocity? Assume that the man's mass is negligible and that his arms have length *d* when extended and are *d*/4 from the center of his body when at his sides.

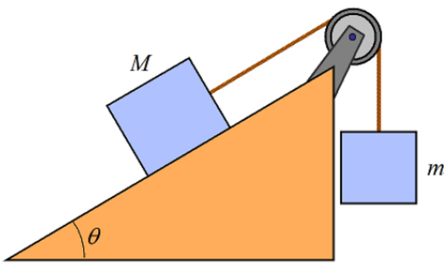
[12] **Bug on the rod:** A massless rod of length 2L is free to rotate about a frictionless axle through its center. At one end of the rod a small bug of mass m is at rest on the top of the rod. At the other end of the rod is a block of mass 4m . The rod initially rotates with an angular velocity of ω_0.

- (a) What is the initial angular momentum of the system?
- (b) If the bug walks towards the center of the rod at a constant rate of v_1 , find the angular velocity of the system as a function of time.
- (c) What is the angular component of the force on the bug as a function of time while it is moving towards the center of the rod?
- (d) Repeat the problem considering the rod has inertia I.



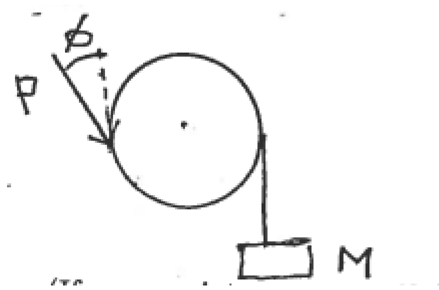
[13] Two disks are spinning on a frictionless axle. The left disk has a mass M , radius R_L , a moment of inertia I_L , and an angular velocity of ω_L in the direction shown. The disk on the right has a mass 2M , radius R_R , moment of inertia I_R , and angular velocity of ω_R in the direction shown. The disks move linearly towards each other with equal speeds of v_0 until they collide. What are the final angular velocity if they stick together after the collision?

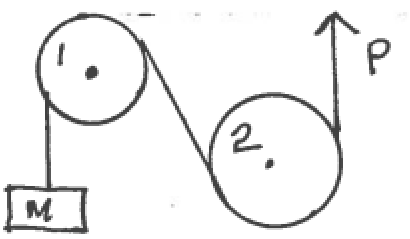
:: Newton's law for rotational motion ::



[14] **Summer,'16:** Two blocks, mass M and m, are connected by a massless, unstretchable string. The string goes over a pulley that has radius R and moment of inertia I about its center. There is no fricion about the axle of the pulley. There is no slipping of the string in contact with the pulley. There is friction between M and the inclined plane, with coefficient of friction μ. Assuming the block of mass m is going down, what will its acceleration be?

[15] **Summer,'19:** A block of mass M is suspended by a massless, unstretchable rope from the pulley as shown. The pulley rotates freely about the a fixed horizontal axle. The rope moves around the pulley without slipping. The pulley has a moment of inertia about its center of I and a radius R. A constant force of magnitude P is applied to the pulley as shown at the known angle ϕ. What will be the acceleration of the block, assuming it moves down? If the hanging mass is released from rest at t=0, what will be the angular velocity of the pulley as a function of time?





[16] **Summer,'18:** A block of mass M is suspended by a massless, unstretchable rope from the pulley system shown. The pulleys rotate freely about fixed horizontal axles. The rope moves around the pulleys without slipping. Pulley 1 has a moment of inertia I_1 and radius r_1. Pulley 2 has a moment of inertia I_2 and radius r_2. If a constant force of magnitude P is applied to the rope as shown, what will be the acceleration of the block?